Histogram-Based Bit Error Ratio Estimator for Differential Modulation Formats

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Abstract: A histogram-based BER estimation method for differential modulation formats is analyzed. It is shown that this BER estimation approach yields more accurate results than conventional extrapolation methods with less simulated symbols.

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1. Introduction

For efficient switching of Ethernet traffic through long haul optical transmission systems, Carrier-Grade Ethernet is a current topic of research [1]. The bit rate per channel that is suggested for this technology is 100 GB/s (100 GE). These channels have to be transmitted within the available frequency slots in existing networks with a 100 GHz spacing. This narrow channel spacing and the bit rate of the channel demand a modulation format with a high bandwidth efficiency. DQPSK is one of the formats that is able to cope with this constraint and shows additionally promising behavior towards other degradation effects.

In order to analyze the behavior of the modulation format, an accurate estimation of the bit error ratio (BER) of the detected signal is necessary. The estimation method we propose in this paper represents a new approach with promising characteristics regarding the necessary number of symbols for the BER estimation and the level of uncertainty of the result. In addition it is, in contrast to other estimation techniques like the semi-analytical Karhunene-Loève expansion [2], also resilient to nonlinear effects like nonlinear phase noise. These characteristics play a significant role in minimizing the computational effort and the level of uncertainty during the simulation of optical transmission systems and are hence important features of BER estimators. The estimation method is based on the separate estimation of the histograms of the mark and space values at the sampling time for both PIN diodes of the balanced receiver. This independent histogram analysis and the following processing of the results yield less uncertain BER values of the transmitted and detected system.

2. Simulation setup

The analyzed transmission system consists of a DQPSK transmitter with a cascaded Mach-Zehnder modulator structure, a single fiber span and the differential receiver structure for both I- and Q-channels. In order to use this BER estimation technique, more information about the received signal is analyzed. In contrast to standard balanced receivers, this receiver type detects the photo current of each PIN diode separately. This is important since histograms for both PIN diodes are required for the following calculation of the BER.

![Fig. 1. Examined DQPSK receiver setup (for one channel)](a1421_1.pdf)
Fig. 1 shows the adapted receiver setup for one of the two differentially encoded channels of the DQPSK signal. Since the only difference between the $I$ and $Q$ channel receivers is the phase offset parallel to the delay element, the following studies deal only with the receiver part for the $I$-channel.

In a conventional receiver setup a bit error ratio tester (BERT) placed at the position of output two is the only component used for measuring the BER. Here this BERT is solely used for generating the reference data with standard Monte Carlo simulations. The data used for the new method originates in eye analyzers at output one and three.

3. **BER estimation method**

Each symbol received by the structure depicted in Fig. 1 is represented by two separately recorded values: the photo current measured at output one and three at the optimum sampling time. In a conventional balanced receiver the difference of these two values is detected used for further calculations. This modified receiver considers these two values independently. For every possible symbol (0/1) two histograms are calculated. To account for the differential characteristics of this receiver, the histograms of output three have to be mirrored at the $y$-axis. Since the level of uncertainty decreases with increasing counts for a given a histogram bin only bins with more than ten events are taken into account.

Due to the computational effort in a numeric simulation the number of symbols available is limited. With histograms generated from this statistics the BER cannot be calculated directly. To estimate the BER outside the range of the histograms, functions are fitted to the histograms. Due to the quadratic behaviour of the PIN diode chi$^2$ distribution functions are better suited as fitting functions than Gaussian approaches [3]. Since no correlation between the noise parts of the two signals could be observed, we assume these signals to be statistically independent. Hence the fitted functions for both PIN diodes can then be convolved as depicted in Fig. 2. The result corresponds to a histogram of the received signal of output two. A functional fit of the histogram of output two would be more difficult due to the fact that no elementary functions such as chi$^2$ could be used.

In order to calculate the actual BER of the system there are several steps necessary. First the convolved histograms have to be normalized to get real probability density functions. Therefore the integral of each convolved histogram has to be equal to unity. By integrating the normalized curves from minus infinity (for a ‘1’ symbol) or plus infinity (for a ‘0’ symbol) to a variable threshold, curves are generated that show the dependency of the BER of each symbol to the threshold. These individual BER curves then have to be weighted with regard to the probability of the appearance of the symbol they represent and added to get the overall BER curve of the signal. The abscissa of the minimum of this curve represents the optimal threshold and the ordinate the according optimal BER value.

![Fig. 2. Histograms of eye analyzers 1 (a) and 3(b) and the convolved result (c) for a ‘0’ symbol ($I$ channel).](image)

4. **Simulation results**

In order to evaluate the performance of the presented BER estimator, its results are compared to the performance of other well known estimation techniques such as the standard Monte Carlo method and the tail extrapolation technique [4]. The graph in Fig. 3 (a) shows the result of these different BER estimation techniques for the same transmission system. The reference method is the standard Monte Carlo method with an optimized decision threshold. In order to decrease the level of uncertainty, only results with more than 100 detected bit errors at this
threshold were considered. As a drawback it prevents that BER values less than $10^{-5}$ can be presented due to the linearly increasing computational effort. To compare also the influence of the function type the histograms are fitted to, also a Gaussian function fit is presented that should be more uncertain according to [5].

Due to the goal of the estimation techniques to enable to reduce the amount of simulated symbols without loss of accuracy, the analyzed BER estimators only take 10,000 symbols into account. One can see that the technique presented in this paper for fits with chi² functions is superior to the common Gaussian tail extrapolation since its predictions are closer to the one of the standard Monte Carlo method with 100 erroneous detected bits. This may stem from the fact that the tail of the distribution of the symbols is no straight line as can be seen in Fig. 2 (c). For high BER values such as $10^{-4}$ that does not yield a significant error, but for BER values as low as $10^{-9}$, the difference increases up to almost an order of magnitude. This shows that histogram based functional fits with chi² approximations are better suited for estimating the BER of a differential receiver types than Gaussian approximation and Gaussian tail extrapolations.

The other interesting characteristic of a BER estimator is the number of symbols that is necessary for the estimator to converge towards its asymptotic value. The behaviour for the different types of BER estimators is shown in Fig. 3 (b). Here it can be seen that the histogram based fit with chi² as the fitting function only needs 10,000 symbols to reach its asymptotic value. In contrast to that the often used tail extrapolation needs approximately twice as many symbols.

5. Conclusion

The estimation technique analyzed in this paper shows superior behavior to that of a standard Gaussian tail extrapolation. The combination of less uncertainty and less required symbols makes the new technique highly interesting for the analysis of differential modulation techniques in numerical simulations.

6. References