Higher Order Polarization-Mode Dispersion in the Presence of Polarization-Dependent Loss in Optical Fiber Systems

Ansgar Steinkamp and Edgar Vöges

Abstract—The statistics of first- and second-order polarization-mode dispersion and statistical dependencies between corresponding quantities are investigated with regard to fiber-optic links with polarization-dependent loss (PDL). We show the occurrence of an infinite depolarization rate of the principal states of polarization (PSPs) caused by an abrupt interchange of the two PSPs with a probability that grows with the global PDL. We finally discuss the influence of the frequency resolution of measurement and simulation methods on the depolarization results.

Index Terms—Optical fiber communication, polarization-dependent loss (PDL), polarization-mode dispersion (PMD).

I. INTRODUCTION

LONG-DISTANCE all-optical links suffer not only from polarization-mode dispersion (PMD) caused by the random birefringence of optical fibers but also from the polarization-dependent loss or gain (PDL/G) of a large number of inline components. In this letter, we study the statistics of PMD in the presence of PDL (or PDG).

Probability densities of first- and second-order PMD have been derived analytically in the past decade assuming the absence of PDL [1]. In the presence of both PMD and PDL, Li and Yariv [2] succeeded in analytically deriving various scalar expectation values. The corresponding probability densities, though, have not been calculated yet. Some of them have been estimated by means of Monte Carlo simulations: The differential group delay $\Delta \tau$ (DGD) [3], [4], the angle between the principal states of polarization (PSPs) in Stokes space $i_{\pm}$ [4], and the magnitude of the second-order PMD-vector [4]. However, to our knowledge, the statistics of the two key parameters of second-order PMD (i.e., the polarization-dependent chromatic dispersion $\partial_{\lambda} \Delta \tau$ (PCD) and the depolarization rate of the PSPs $|\partial_{\lambda} i_{\pm}|$) and their statistical dependencies have not been carefully addressed so far in the presence of PDL.

Since the mean DGD $\langle \Delta \tau \rangle$ scales all quantities of PMD and combined PMD and PDL as well as their change with frequency [4]–[6], we present all results in normalized units (normalized by $\langle \Delta \tau \rangle$ and $\langle \Delta \tau \rangle^2$, respectively). Thus, our results can be applied to links with any amount of PMD. In consequence, the frequency $\omega$ is normalized by the bandwidth of the principal states $\Delta \omega_{\text{PSP}} \equiv (\pi/4)/\langle \Delta \tau \rangle$.

II. MODEL AND RESULTS

The transmission matrix of a linear optical medium $T(\omega)$ is not unitary in the presence of PDL ($T^{-1} \neq T^\dagger$). Linking our notation to [1] and [7], $T$ relates input and output Jones vectors by $|T\rangle = T|\psi\rangle$. Neglecting polarization-independent losses (which can be factored out without influencing the analysis), Gisin and Huttner have shown that the matrix $2\sqrt{\eta} T^{-1}$ is trace-less even in systems with PDL [5]. (The subscript indicates differentiation, $\eta$ is the imaginary unit.) Since the (permutated) Pauli spin matrices $\sigma_1, \sigma_2,$ and $\sigma_3$ [7] form a basis of $2 \times 2$ matrices with zero trace, $2\sqrt{\eta} T^{-1}$ can be expanded in the form $2\sqrt{\eta} T^{-1} = \gamma_1 \sigma_1 + \gamma_2 \sigma_2 + \gamma_3 \sigma_3 \equiv \mathcal{F} \cdot \mathcal{D}$. The coefficients define the PMD vector $\mathcal{F}$ [5]. In the absence of PDL, the matrix $2\sqrt{\eta} T^{-1}$ is Hermitian [7], $2\sqrt{\eta} T^{-1} = (2\sqrt{\eta} T^{-1})^\dagger$, which causes the PMD vector to be a real vector in Stokes space. This condition does not hold in systems with PDL, turning the PMD vector into a complex vector in that case [5].

The existence of PSPs (polarization states for which the output polarization is independent of frequency to first order) even in systems with PMD and PDL has been proved in [5]. The output PSPs in Jones space $|t_{\pm}\rangle$ turn out to be the eigenvectors of $2\sqrt{\eta} T^{-1}$ with complex eigenvalues whose real part is the DGD $\Delta \tau$:

$$2\sqrt{\eta} T^{-1} |t_{\pm}\rangle = \pm (\Delta \tau + i\Delta \eta) |t_{\pm}\rangle.$$  

The geometric interpretation of the imaginary part $\Delta \eta$ (and of the complex PMD vector $\mathcal{F}$ and its dynamical equation) can be found in [5] and [2]. Since $2\sqrt{\eta} T^{-1}$ is not Hermitian in systems with PDL, the eigenvalues are not real and the eigenvectors are not orthogonal anymore. This means that the PSPs in Stokes space $i_{\pm} \equiv (t_{\pm} |\mathcal{F}\rangle |t_{\pm}\rangle)$ are no longer antipodal.

Based on [5], the transmission link in our simulations is modeled as a concatenation of birefringent elements randomly oriented in Stokes space with PDL elements randomly inserted along the link and randomly oriented. The number of PDL elements is chosen to be 200 (corresponding to a long-haul system with 40 spans with around five PDL elements at each amplifier location).\textsuperscript{1}

Our analysis starts with the calculation of the transmission matrix $T(\omega)$ with high spectral resolution $\Delta \omega = \Delta \omega_{\text{PSP}}/500$.

\textsuperscript{1}Further details of our system: Number of DGD elements: 8000; ensemble average of the global DGD in the absence of PDL: $\langle \Delta \tau \rangle = 4\pi \Rightarrow \Delta \omega_{\text{PSP}} = 2\pi \cdot 31.25 \text{ GHz}$. However, neither the exact number of DGD elements nor the value of the average DGD nor the fact that the elements’ directions are not bound to the equator of the Poincare sphere, is of any relevance to the statistics of our results. Furthermore, all our results can be applied to systems with a smaller number of PDL elements, too: Dividing the number of PDL elements by $N$ while multiplying the individual PDL by $\sqrt{N}$ leads (approximately) to statistically equivalent results.

Manuscript received September 22, 2006; revised November 18, 2006. This work was supported in part by T-Systems Enterprise Services GmbH, Germany.

The authors are with the High Frequency Institute, University of Dortmund, D-44227 Dortmund, Germany (e-mail: ansgar.steinkamp@uni-dortmund.de).

Digital Object Identifier 10.1109/LPT.2006.890090
This enables us to evaluate $\mathbf{T}_{\Delta \tau}$ (by numerical differentiation), the matrix $2\mathbf{IT}_{\Delta \tau}\mathbf{T}_{\Delta \tau}^{-1}$, the PSPs in Jones $\mathbf{I}_{\Delta \tau}$ and Stokes space $\mathbf{I}_{\Delta \tau}$, and the DGD $\Delta \tau$. Finally, the depolarization rate of the PSPs $|\mathbf{I}_{\Delta \tau}^x|\mathbf{I}_{\Delta \tau}^y$ and the PCD $\partial_\tau \Delta \tau$ are calculated by numerical differentiation.

Fig. 1 shows the ensemble statistics of the link. As can be seen from Fig. 1(a), large values of PDL cause an increased probability of very high and very low instantaneous DGD. (The former leads to an increased outage probability of the system.) There is only a small effect on its average, though. The PSPs are no longer antipodal in Stokes space [Fig. 1(b)] even if the PDL is rather low. This generates several peculiar effects on signal transmission [5], [6]. Fig. 1(c) and (d) shows the effect of PDL on PCD and PSP depolarization. Large PCD-events in the tail of the distribution occur much more frequently in systems with large PDL, whereas the probability density of PSP depolarization remains rather stable, even if the individual PDL is as high as 0.8 dB. ²

Statistical dependencies between the DGD and the PSP depolarization rate in the absence of PDL have already been mentioned in [9] and [1]. Their results agree with the ones of the ensemble in Fig. 3 con

By taking a closer look at the simulation results in the presence of PDL, we identify a very small probability of extremely high PSP depolarization $\gg 10$ with an upper limit of $\approx \gamma_{37}$ (in normalized units). A further decrease of the step size below $\Delta \omega = \Delta \omega_{\text{PSP}}/500$ reduces the occurrence probability and increases the upper limit. The investigation of PSP depolarization as a function of frequency (of a randomly chosen realization of the ensemble) in Fig. 3 confirms these observations: Spikes of PSP depolarization [9], [1] tend to grow to infinity in the presence of PDL (in the limit $\Delta \tau \rightarrow 0$). Fig. 4 shows the PSPs on the Poincare sphere in the vicinity of the normalized frequency $\approx 6.13$ of Fig. 3. At this frequency, the first transition of a depolarization spike to infinite depolarization occurs if the PDL is increased from 0.1 to 0.2 dB. A rapid change of the PSPs’ direction in Fig. 4(a) turns into an abrupt interchange of the PSPs in Fig. 4(b). This sudden change of direction of the PSPs translates into infinite PSP depolarization. An extended analysis of the statistics of depolarization shows that the occurrence probability of infinite PSP depolarization in the bandwidth $\Delta \omega_{\text{PSP}}$ is $\approx 3\%$ at $200 \times 0.8$ dB PDL. This decreases to $\approx 0.5\%$ at $200 \times 0.4$ dB.
The calculation of PSP depolarization $|\partial_\omega \tilde{t}_{\pm}|$ in practice is based on numerical differentiation of measured (or simulated) high-resolution data of the PSPs in Stokes space $\tilde{t}_{\pm}(\omega)$. This is why the occurrence of infinite PSP depolarization leads to a measurement result, which strongly depends on the step size $\Delta \omega \equiv \omega_{n+1} - \omega_n$ of the data. With $|\partial_\omega \tilde{t}_{\pm}(\omega_n)| = |\tilde{t}_{\pm}(\omega_{n+1}) - \tilde{t}_{\pm}(\omega_{n-1})|/(2\Delta \omega)$ and the angle between the PSPs in Stokes space $\theta_{\text{PSP}}$, the result of a measurement (or simulation) at a frequency of infinite PSP depolarization is straightforward to calculate

$$|\partial_\omega \tilde{t}_{\pm}| = \frac{\Delta \omega_{\text{PSP}}}{\Delta \omega} \sin \left(\frac{\theta_{\text{PSP}}}{2}\right).$$

This is in excellent agreement with the data of Fig. 3 and the upper limit we mentioned in connection with Figs. 1 and 2.

In Fig. 5, we take a closer look at the transition region of Fig. 3(a) and (b). The DGD $\Delta \tau$ in Fig. 5(b) vanishes at the frequency of infinite PSP depolarization with an abrupt change of slope. This leads to the change of sign and magnitude of the PCD in Fig. 5(d) and is responsible for the modified scatter plot of Fig. 2(d). The angle between the PSPs in Stokes space is much less than $180^\circ$ in the vicinity of this frequency [Fig. 5(c)]. Not shown in Fig. 5 is the magnitude of the (generalized) second-order PMD vector $|\partial_\omega (\Delta \tau \cdot \tilde{t}_{\pm})|$, which is increased by $\approx 30\%$ in the transition region ($0.2$ dB compared to $0.1$ dB) and has a negligible discontinuity at the frequency of infinite PSP depolarization. Since the DGD is very small at this frequency, the impact of the discontinuities on the distortion of the signal and on the efficiency of optical PMD compensators might be limited. This is presently under study and we intend to publish the results elsewhere.

III. CONCLUSION

We have analyzed the influence of PDL on the statistics of PMD and statistical interdependencies between first- and second-order PMD. The investigation of the frequency dependence shows that infinite PSP depolarization, infinitesimally small DGD, highly nonorthogonal PSPs, and large PCD tend to occur simultaneously.

REFERENCES


