Experimental Verification of Fast Analytical Models for Four-Wave Mixing (FWM)-Impaired Transparent Optical Networks

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ABSTRACT

In this paper four-wave mixing (FWM) and its impact on multi-span NRZ-modulated wavelength division multiplexing (WDM) systems are examined. The impact of the inline dispersion compensation map on FWM impairments is crucial. From re-circulating loop experiments as well as our analytical model worst-case conditions for the dispersion map are found. Furthermore, the impact of increasing the number of WDM channels is investigated. An analytical model is presented to assess the signal degradation. The impairments due to FWM are related to a Q-factor or an EOP. The presented formulas are applied to different dispersion compensation schemes and also mixed-fiber systems. The analytical model is verified by system simulations employing the split-step Fourier method as well as re-circulating loop experiments.

Keywords: four-wave mixing, nonlinear fiber optics, optical crosstalk, optical fiber communications, wavelength division multiplexing.

1. INTRODUCTION

Four-wave mixing (FWM) is one of the dominant degradation effects in 10 Gb/s NRZ WDM systems with dense channel spacing and low chromatic dispersion fiber. If in a WDM system the channels are equally spaced, the new waves generated by FWM will fall at channel frequencies and thus will give rise to crosstalk. This phenomenon leads to intermodulation effects, which can limit the minimal channel spacing and the overall capacity of the system. To reduce the phase matching in the fibers, non-zero dispersion shifted fibers have been developed, which have the dispersion zero outside of the transmission band. However, if the system is fully loaded, several channels will lie at the blue edge of the C-band, where the local dispersion may still be very low. In this paper the signal quality of an FWM-impaired optical network is assessed analytically as well as experimentally. Attention is payed to the inline dispersion compensation map. It is shown that distributed undercompensation of the dispersion does not necessarily lead to better FWM suppression. The analytical model used in this paper is based on a cw-approximation [1][2]. In future automatically switched optical networks it will be mandatory to assess the signal quality of a new path instantaneously. The routing has to take into account the physical constraints of the network. The analytical models used in this paper are suited to this task because they are computationally efficient and very fast.

2. ANALYTICAL MODEL FOR THE ASSESSMENT OF FWM

Four-wave mixing (FWM) is a third-order nonlinearity analogous to intermodulation distortion in electric systems. The origin of FWM lies in the power-dependence of the refractive index of the fiber. The name four-wave mixing comes from the fact that three waves of the frequencies \( f_i, f_j, f_k \) \((k \neq i, j)\) give rise to a fourth wave through interaction with the third-order susceptibility [1].

\[
f_{ijk} = f_i + f_j - f_k
\]  

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The inter-channel FWM effect is in a first approximation independent of the bit rate, but crucially dependent on the channel spacing and the local dispersion. If in a WDM system all channels have the same channel spacing, the newly generated waves will fall at signal frequencies and will lead to crosstalk. The amplitude of a FWM mixing product at frequency $m$, which is generated by waves $i$, $j$ and $k$, can be calculated in the general case from the following equation [1]:

$$A_{m,ijk}(l_1, l_2) = j\gamma \int_{z_{l_1}}^{z_{l_2}} A_i(0) A_j(0) A_k(0) \exp(-\alpha z - j\Delta\beta_{ijk} z) dz$$  \hspace{1cm} (2)

Where $A$ is the amplitude of the envelope of the different waves, $\gamma$ is the nonlinearity constant of the fiber and $\alpha$ the attenuation constant. $l_1$ and $l_2$ are the absolute positions on the fiber, measured from the beginning of the transmission system. The degeneracy factor $d$ has a value of 3 for $i = j$ (two-tone product), resp. 6 for $i \neq j$ (three-tone product). In eq. (2) the amplitude distortion due to dispersion has been neglected. This is valid for low-dispersion fibers such as DSF or NZDSF and moderate bit rates (i.e. 10 Gb/s systems), where the FWM degradation becomes maximal. The phase matching between the different waves can be calculated from the following formula [3]:

$$\Delta\beta = \beta_i + \beta_j - \beta_k - \beta_{ijk} = \frac{2\pi^2}{c_0} (f_i - f_j)(f_j - f_i) D(f_j) - \frac{\Delta^2}{c_0} \left( \frac{f_i + f_j}{2} - f_i \right) S$$  \hspace{1cm} (3)

With the dispersion coefficient $D$ [ps/(nm⋅km)] and the dispersion slope $S$ [ps/(nm²⋅km)]. If the phases of the different four-wave mixing products are assumed independent, the powers of the FWM products at a certain frequency can be summed. To a good approximation the optical phases are decorrelated due to the dispersive propagation in the fiber. In contrast to the cw-case, the probability of the FWM products has to be taken into account for modulated signals. For pseudo-random bit sequences (PRBS), the number of ones and zeros is balanced. This leads to the probability of a three-tone product ($f_i \neq f_j$), which is 1/8, and the probability of a two-tone product ($f_i = f_j$), which is 1/4. In the case of a high extinction ratio, the mean value of the zeros is very low and also the FWM-induced variance of the zeros can be neglected. In this case a Q-factor can be calculated from the following equation:

$$Q = \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} \hspace{1cm} \text{with} \hspace{1cm} \sigma^2 = 2 P_{\text{channel}} \sum_{\text{ijk}} P_{m,ijk}$$  \hspace{1cm} (4)

Where $\mu_1$ and $\mu_0$ are the mean values of the one lines and the zeros lines, respectively, $\sigma_1$ and $\sigma_0$ are the corresponding standard deviations and $P_{m,ijk}$ is the FWM power at the position $m$ generated by channels $i$, $j$ and $k$. The Q-factor assessment in eq. (4) assumes a Gaussian distribution of the distorted eye lines. In [4] this has been confirmed for a high number of mixing products (>10). Furthermore, it has been assumed that $P_{\text{channel}} >> P_{m,ijk}$, where $P_{\text{channel}}$ is the mean power of the signal channel. By this assumption the additional noise term, which is proportional to $P_{m,ijk}^2$, can be neglected. Therefore a Gaussian- instead of the more complex Rician distribution can be used [5]. The Q-factor in eq. (4) does not take into account the ASE noise. By summing the noise variances due to FWM- and ASE-noise, a Q-factor can be derived easily, which takes into account both effects. In the following a high OSNR has been assumed so that the degradations due to ASE can be neglected compared to the FWM-induced variance.

The FWM power $P_{m,ijk}$ used in eq. (4) can be determined by the following equation.

$$P_{m,ijk}(L) = |A_{m,ijk}(L)|^2 = |\bar{A}_{m,ijk}(L)|^2 e^{-\alpha L}$$  \hspace{1cm} (5)
For a low number of signal channels it is advantageous to use an EOP instead of a Q-factor because the distribution of the eye lines is not Gaussian (compare Fig. 1). In this case the EOP can be calculated from the following equation (eq. 6), which has been originally derived for linear crosstalk between two neighboring signal channels. The FWM power leads to a mixing term, which decreases the amplitude of the undistorted signal channel.

\[
EOP = 10 \log \left( 1 - 2 \sqrt{\frac{P_{FWM}}{P_{signal}}} \right)
\]  

(6)

For special cases of the inline dispersion compensation such as the full-inline dispersion compensation (FOCS [6]), it is possible to derive an analytical equation for the overall four-wave mixing power generated by all signal frequencies [11]. The assumptions taken in the following equation are: the input power of all channels is constant, the channel spacing is constant and the dispersion is compensated 100% per span. The local dispersion of the fiber has been assumed to be non-zero (i.e. NZDSF or SSMF are employed).

\[
P_{\text{out,FWM}} = \gamma^2 P^4 \left[ (1 - e^{-\alpha \Delta f})^2 + 4e^{-\alpha \Delta f} \right] \frac{W_T}{(4\pi^2 \beta_2 \Delta f)^2}
\]  

(7)

where \( \beta_2 \) [ps²/km] is the GVD parameter and the channel spacing is denoted by \( \Delta f \). Eq. (7) gives a worst-case approximation. The polarization of all channels is assumed linear and parallel. The signal degradation is assessed at the center channel position, which represents the worst-case for FOCS. The weighting factor \( W_T \) can be calculated from the following equation:

\[
W_T = \sum_{i,k} \left( \frac{d}{3 \cdot (i-k)(j-k)} \right)^2 = -7.13 + 22.22(1 - e^{-\frac{N}{4.57}}) + 4.05(1 - e^{-\frac{N}{31.01}})
\]  

(8)

In eq. (8) \( N \) defines the total number of WDM channels. \( d \) is the degeneracy factor (3 for \( i = j \), 6 for \( i \neq j \)). The approximation formula in eq. (8) has been derived by fitting an exponential function to the summation formula.

From eq. (8) also the qualitative behavior for an increase of the channel number can be derived. For a channel count of more than 10, the increase in signal degradation is relatively small (compare Fig. 2). This behavior can be explained as follows. Any signal channels added at the edge of the signal spectrum have a relatively large channel spacing to the regarded center channel. Due to the local dispersion on the fiber, the new channels have a high phase mismatch to the center channel and the FWM efficiency is very low.
3. EXPERIMENT SETUP

For the experiment the following re-circulating loop setup has been used (Fig. 3). Twenty NRZ-modulated 9.953 Gb/s channels have been spaced on a 50 GHz grid. All channels had a parallel polarization at the loop input, which represents the worst-case situation.

In the experiment a pre-DCF has been employed to decorrelate the channels. Amplifiers were located before each transmission fiber and each DCF. Inside the loop three spans were cascaded. An average distributed undercompensation (DUCS [6]) of -50 ps/nm/span was selected for the three spans incorporated in the loop (s. Table 1). Alternatively full-inline compensation (FOCS [6]) has been used (0 ps/nm in the first span, -6 ps/nm in the second span and 0 ps/nm in the third span). No additional post-compensation was employed at the output of the loop because this does not affect the signal distortions due to FWM. The booster amplifiers had an output power of 1 dBm/ch for all transmission fibers and -2 dBm/ch for the dispersion compensating fibers (DCF). The span lengths were 68 km, 68 km and 64 km, respectively. The fiber parameters of the experiment are shown in Table 1.

<table>
<thead>
<tr>
<th>NZDSF Spans 1; 2; 3</th>
<th>DCF (DUCS) Spans 1; 2; 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ [dB/km]</td>
<td>0.21</td>
</tr>
<tr>
<td>$D$ [ps/nm]</td>
<td>169 (1st); 177 (2nd); 156 (3rd)</td>
</tr>
<tr>
<td>$\delta$ [ps/nm²]</td>
<td>4.6 (1st); 4.5 (2nd); 4.17 (3rd)</td>
</tr>
<tr>
<td>$\gamma$ [1/(W·km)]</td>
<td>2.18</td>
</tr>
</tbody>
</table>

Table 1: Fiber parameters ($\lambda = 1535$ nm).
4. DISPERSION COMPENSATION MAPS

In case of full-inline dispersion compensation, i.e. 100% dispersion compensation per span, the FWM crosstalk becomes maximum since the FWM products add coherently in each span [7]. Here we demonstrate both analytically and experimentally that also other inline dispersion maps result in a worst-case for FWM-impaired systems.

In [7] it has been shown that the FWM mixing power scales with the following factor:

$$ W_{FWM} = \left| \sum_{n=1}^{M} \exp\left(-j(m-1)\Delta\psi\right) \right|^2 = \frac{\sin^2\left(\frac{M \Delta\psi}{2}\right)}{\sin^2\left(\frac{\Delta\psi}{2}\right)} $$

(9)

$M$ defines the total number of spans, $\Delta\psi$ involves the propagation constant of the FWM products and is directly related to the amount of inline residual dispersion per span:

$$ \Delta\psi = \Delta\beta_{NZDSF} \cdot L_{NZDSF} + \Delta\beta_{DCF} \cdot L_{DCF} $$

$$ \equiv D_{NZDSF} \cdot L_{NZDSF} + D_{DCF} \cdot L_{DCF} $$

(10)

The phase mismatch between the different channels is described by $\Delta\beta$. Eq. (9) results in a worst-case scenario for the commonly used distributed undercompensation scheme (DUCS [6]) of –50 ps/nm/span because in this case the denominator gets very small. The optimum can be achieved for an undercompensation of approx. –40 ps/nm/span when the numerator becomes very small.

In the experiment the FWM power was measured outside of the signal spectrum with an optical spectrum analyzer (OSA). For this purpose the center channel was omitted so that at this position the FWM power could be measured (s. inset Fig. 5). The FWM power is considered as a figure of merit for the signal degradation because it will lead to crosstalk whenever it falls directly onto a signal channel. In Figs. 4 and 5 the ratio of the signal peak power to the FWM peak power is plotted on the ordinate.

For an undercompensation of –50 ps/nm/span it is shown that the increase of the FWM power with span count is comparable to the full-inline compensation case (compare Figs. 4 and 5). Note that this behavior is found for an average undercompensation of –50 ps/nm/span. The amount of undercompensation does not necessarily exactly meet this value in each span. Also the span length may vary on a span-by-span basis. Only the absolute amount of undercompensation per span is the relevant parameter. In Fig. 4 the dependence of the FWM mixing power on the inline dispersion compensation map is shown, whereas in Fig. 5 experimental and analytical results have been compared for two dispersion compensation maps, namely –50 ps/nm/span and full-inline compensation in each span.

Fig. 4: Comparison of different dispersion compensation maps (analytical results)
1 dBm/ch, NZDSF ($D = 2.4$ ps/(nm·km), $S = 0.06$ ps/(nm²·km)), 50 GHz channel spacing, 21 channels.
Fig. 4 also shows the values calculated from the analytical equation (eq. (2)) for comparison. It can be seen that an undercompensation of $-40 \text{ ps/nm/span}$ shows a clear advantage compared to other inline residual dispersion values. The discrepancy between the analytical curves and the measured one in Fig. 5 stems mainly from the fact that the DCFs did not match the exact FOCS and DUCS compensation maps, but varied slightly. Also the DCF nonlinearity was neglected in the analytical model.

Fig. 5: Measured ratio of the signal power to the FWM power of the center channel ($\lambda_0 = 1535 \text{ nm}$) for a system consisting of 20 NRZ channels, 1 dBm/ch, NZDSF ($D_{\text{acc}} = 169 \text{ ps/nm}, S_{\text{acc}} = 4.6 \text{ ps/nm}^2$) fibers with DUCS and FOCS. For comparison analytical results are included. The inset shows the spectrum measured with an OSA. The OSNR was measured as the ratio between the signal power and the FWM power measured at the center position. Furthermore it is important to mention that the worst-case degradation in distributed-undercompensation maps does not necessarily occur for the center channel as for the FOCS case. Fig. 6 shows the distribution of the FWM-induced OSNR over the spectrum for an FWM-impaired system. For the FOCS case the OSNR shows a parabolic structure. In contrast the FWM-induced OSNR value is oscillating in the DUCS case with the channel frequency. The total amplitude also varies significantly with the dispersion compensation map. For experiments it is important to mention that solely measuring the signal quality at the center channel position is not sufficient for assessing the overall signal quality of a system and does not necessarily represent the worst-case. However, if the dispersion compensation map is not perfectly homogeneous and a mismatch due to the dispersion slope is assumed, the amplitude of the OSNR oscillations will be much less pronounced than depicted in Fig. 6.

Fig. 6: Distribution of the FWM-induced OSNR over the spectrum (analytical results), 9 channels, 3 dBm/channel, $D = 4.9 \text{ ps/(nm\cdot km)}$, $S = 0 \text{ ps/(nm}^2\text{-km)}$, 50 GHz channel spacing. The OSNR is plotted after 24 spans.
Furthermore, the number of WDM channels and the input power into the NZDSF have been varied in the experiment. As can be seen from Fig. 7 the influence of the number of WDM channels is twofold. First of all the FWM-induced OSNR is decreasing for a higher number of WDM channels. This is due to the higher number of FWM products falling in the regarded center channel considered in Fig. 7. Also the statistics is better in the 20 channels case. You can clearly see that the OSNR line shows an oscillatory structure for 10 channels, whereas it resembles a straight line in the 20 channel case.

![Graph showing the influence of the number of WDM channels on OSNR.](image)

**Fig. 7:** Experimental FWM-induced OSNR of the center channel ($\lambda_0 = 1535$ nm) for a system consisting of 10/20 NRZ channels, NZDSF ($D_{acc} = 169$ ps/nm, $S_{acc} = 4.6$ ps/nm²) fibers with DUCS -100 ps / (nm-span) (1 dBm/ch).

If the input power is varied, it can be seen very nicely that the curves nearly run in parallel (Fig. 8). The higher the input power, the lower the Q-factor. As can be seen from eq. (2) and (5), the power of the FWM mixing products scales with the third power of the input channel power.

![Graph showing the influence of input power on OSNR.](image)

**Fig. 8:** Experimental FWM-induced OSNR of the center channel ($\lambda_0 = 1535$ nm) for a system consisting of 10 NRZ channels, NZDSF ($D_{acc} = 169$ ps/nm, $S_{acc} = 4.6$ ps/nm²) fibers with DUCS -100 ps / (nm-span) (1 dBm/ch).
Another interesting case is the one of mixed-fiber systems [8]. In today’s deployed network infrastructure it is very common that different fiber types may be included in a long-haul network with a high span count. Especially dispersion shifted fibers (DSF) will lead to a very high degradation due to nearly perfect phase-matching of the different WDM channels. This is why a system, which contains DSF, is preferably operated in the L-band. In the 1580 nm region DSF at least show a small amount of local dispersion, which is approx. 1 ps/(nm-km). In the system considered in this paper DSF have been deployed in the 4th, 10th, 16th and 22th span. All other spans were based on TWRS fiber. An input power of 3 dBm/ch has been used for all spans and 9 channels have been placed on a 50 GHz grid. A 10 Gb/s NRZ modulation scheme has been assumed, which is common for today’s deployed transparent optical networks. A distributed undercompensation scheme has been utilized, which is common for today’s deployed transparent optical networks. A distributed undercompensation scheme has been utilized with -20ps/(nm-span). As can be derived from eq. (9) such an undercompensation is advantageous for suppressing the FWM mixing products, which is especially desirable for highly FWM-impaired systems. At the end of the system the residual dispersion has been tuned to zero. The reason for this was that in the system simulations only the effects of FWM and dispersion have been considered. To eliminate the dispersion impairments from the analyzed signal the dispersion was fully compensated after the last span. This additional post-compensation only ameliorates the eye distortions due to GVD. The power of the FWM mixing product is unaffected by this post compensation. In the split-step Fourier (SSF) simulations [9][10] the Q-factor has been derived according to eq. (4) from the ratio of the mean value and standard deviation of the marks measured with an eye analyzer. The mean-value and the standard deviation were averaged over 100 samples in the center of the eye. Solely the nonlinearity stemming from FWM has been considered in the coupled nonlinear Schrödinger equation. From Fig. 9 it can be derived that the signal quality significantly drops after the 4th span, which incorporates a DSF. The overall system performance is limited by this span. Further DSF spans do not impose a significant additional impairment. Although the FOCS curve is obtained for a pure TWRS system it shows worse impairments than the mixed fiber system with DUCS. For both setups the analytical model shows excellent agreement with the values obtained from SSF simulations (compare Fig. 9). The impact of the dispersion compensation map is crucial for the system performance. This becomes more pronounced for transmission systems, which suffer from a high FWM degradation due to low chromatic dispersion fibers. However, as can be derived from Fig. 9 the overall performance of a system incorporating DSF may be better than for a pure NZDSF system if the dispersion compensation map is chosen properly [11].

Fig. 9: Q-factor of the center channel (\(\lambda_0\) =1570nm) for a mixed fiber system consisting of 9 NRZ channels, TWRS (\(D = 4.9\) ps/(nm-km), \(S = 0.045\) ps/(nm-km²)) and DSF (\(D = 1.12\) ps/(nm-km), \(S = 0.056\) ps/(nm-km²), dispersion values are given for \(\lambda = 1570\) nm) fibers with DUCS -20 ps / (nm-span) (3 dBm/ch) and a pure TWRS system with FOCS (also 3 dBm/ch).
6. CONCLUSIONS

In this paper we have presented a fast analytical model for the analysis of the degradation effects due to FWM. The FWM degradation has been related to a Q-factor or an EOP. The analytical model is suited to the needs of constraint-based routing because it is very fast.

We investigated the impact of varying amounts of inline dispersion compensation. We demonstrated that distributed undercompensation does not necessarily show a significant improvement compared to full-inline compensation. Conditions for worst-case scenarios have been identified. It has been indicated that such a scenario is represented for an undercompensation of –50 ps/nm/span. This case is almost as bad as full inline dispersion compensation, where the amplitudes of all FWM products add-up in phase. This finding is especially important because an undercompensation of about –50 ps/nm/span optimizes SPM- and XPM-impaired systems. However, for systems with very low dispersion fibers, which are FWM-limited, other inline-residual dispersion values should be chosen (e.g. an undercompensation of –40 ps/nm/span).

Furthermore, we have investigated mixed-fiber systems. We have shown that the major part of the overall degradation arises from the first DSF incorporated in the system, if the system is operated in the L-band. For these systems the choice of a good dispersion compensation map is crucial to obtain a reasonable overall system performance.

Experimentally the results from the analytical equations have been confirmed. Excellent agreement between the analytical model and full system simulations using the split-step Fourier method has also been achieved.

REFERENCES